

PAPER-1 (B.E. / B.TECH)

QUESTIONS & SOLUTIONS

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18 March, 2021

03:00 pm to 06:00 pm

SHIFT-2



Duration : 3 Hours

Max. Marks : 300

SUBJECT - MATHEMATICS

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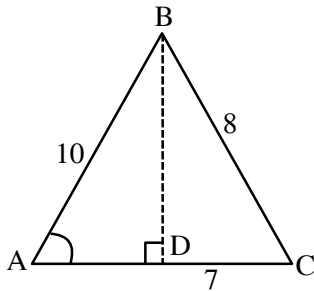
MATHEMATICS

1. In a ΔABC $|\overline{AB}| = 10$, $|\overline{BC}| = 8$, $|\overline{CA}| = 7$, then projection of \overline{AB} on \overline{AC} is

- (1) $\frac{72}{73}$ (2) $\frac{85}{73}$ (3) $\frac{85}{14}$ (4) $\frac{14}{85}$

Ans. (3)

Sol. $\cos A = \frac{10^2 + 7^2 - 8^2}{2 \cdot 10 \cdot 7}$



Projection of AB on AC = $10 \cos A$

$\Rightarrow 10 \cdot \frac{85}{140} = \frac{85}{14}$

2. If $15 \sin^4 \theta + 10 \cos^4 \theta = 6$ then find the value of $8 \operatorname{cosec}^6 \theta + 27 \sec^6 \theta$

- (1) 150 (2) 250 (3) 350 (4) 450

Ans. (2)

Sol. $15 \sin^4 \theta + 10 \cos^4 \theta = 6$

$\Rightarrow 15 \sin^4 \theta + 10 (1 - \sin^2 \theta)^2 = 6$

$\Rightarrow 25 \sin^4 \theta - 20 \sin^2 \theta + 4 = 0$

$\Rightarrow (5 \sin^2 \theta - 2)^2 = 0 \Rightarrow \sin^2 \theta = \frac{2}{5}, \cos^2 \theta = \frac{3}{5}$

Now $27 \operatorname{cosec}^6 \theta + 8 \sec^6 \theta = 27 \left(\frac{125}{27}\right) + 8 \left(\frac{125}{8}\right) = 250$

3. In a series of '2n' observation, half are (a) and other half are (-a) if 'b' is added in all the observation then the mean and S.D. of new series are '5' and '20' respectively then find the value of $a^2 + b^2$

- (1) 450 (2) 480 (3) 475 (4) 425

Ans. (4)

Sol. Given series
(a, a, a n times), (-a, -a, -a n times)

$$\text{now } \bar{x} = \frac{\sum x_i}{2n} = 0$$

as $x_i \rightarrow x_i + b$

then $\bar{x} \rightarrow \bar{x} + b$

$$\text{So, } \bar{x} + b = 5 \Rightarrow b = 5$$

no. change in S.D. due to change in origin

$$\sigma = \sqrt{\frac{\sum x_i^2}{2n} - (\bar{x})^2} = \sqrt{\frac{2na^2}{2n} - 0}$$

$$20 = \sqrt{a^2} \Rightarrow a = 20$$

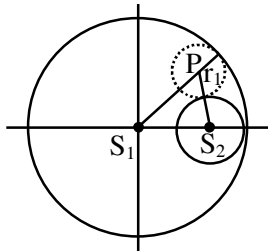
$$a^2 + b^2 = 425$$

4. Let the circle are $S_1 : x^2 + y^2 = 9$, $S_2 : (x - 2)^2 + y^2 = 1$. The locus of the center of the circle which touches S_1 internally and S_2 externally always passes through a point

- (1) (3, 0) (2) $(\sqrt{3}, 1)$ (3) (4, 0) (4) (-3, 0)

Ans. (1)

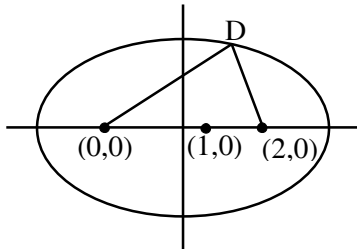
Sol. $PS_1 = 3 - r_1$



$$PS_2 = 1 + r_1$$

$$PS_1 + PS_2 = 4$$

so locus is ellipse & $2a = 4 \Rightarrow a = 2$ & $2ae = 2$



$$\text{Equation of Ellipse is } \frac{(x-1)^2}{2^2} + \frac{y^2}{3} = 1 \Rightarrow e = \frac{1}{2}$$

$$\Rightarrow b^2 = 3$$

(3, 0) satisfied it

5. If $g(x) = \int_0^x f(t)dt$ a here $f(x)$ is a continuous function on $[0,3]$ such that $\forall x \in [0,1]$, $f(x)$ has ranges $\left[\frac{1}{3}, 1\right]$ & $\forall x \in (1,3]$, $f(x)$ has ranges $\left[0, \frac{1}{2}\right]$. Then the maximum range in which $g(3)$ can lie is
- (1) $\left[\frac{1}{3}, 2\right]$ (2) $(1,2)$ (3) $\left[\frac{5}{6}, 3\right]$ (4) $\left[1, \frac{4}{3}\right]$

Ans. (1)

Sol. $\int_0^1 \frac{1}{3} dt + \int_1^3 0 dt < g(3) < \int_0^1 1 dt + \int_1^3 \frac{1}{2} dt$

$$\frac{1}{3} < g(3) < 2$$

6. An experiment is performed 5 times. The probability of 1 success is 0.4096 & probability of 2 success is 0.2048, find the probability of 3 success.

- (1) $\frac{32}{125}$ (2) $\frac{16}{125}$ (3) $\frac{16}{25}$ (4) $\frac{32}{225}$

Ans. (1)

Sol. ${}^5C_1 \cdot p^1 \cdot q^4 = 0.4096$

$$\Rightarrow 5pq^4 = 0.4096 \quad \dots\dots(i)$$

$${}^5C_2 \cdot p^2 \cdot q^3 = 0.2048$$

$$\Rightarrow 10p^2q^3 = 0.2048 \quad \dots\dots(ii)$$

$$(i) \div (ii) \Rightarrow \frac{q}{2p} = 2 \Rightarrow q = 4p$$

$$p + q = 1 \Rightarrow p = \frac{1}{5}, q = \frac{4}{5}$$

$$p(3 \text{ sum}) = {}^5C_3 \left(\frac{1}{5}\right)^3 \cdot \left(\frac{4}{5}\right)^2 = 10 \times \frac{1}{125} \times \frac{16}{25} = \frac{16 \times 2}{125} = \frac{32}{125}$$

7. Let S_n denotes sum of first 'n' terms of an A.P. such that $S_{4n} - S_{2n} = 1000$, then the value of S_{6n} is

- (1) 1000 (2) 3000 (3) 5000 (4) 7000

Ans. (2)

Sol. $S_{4n} - S_{2n} = 1000$

$$\Rightarrow \frac{4n}{2} (2a + (4n - 1)d) - \frac{2n}{2} (2a + (2n - 1)d) = 1000$$

$$\Rightarrow 2an + 6n^2d - nd = 1000$$

$$\Rightarrow \frac{6n}{2} (2a + (6n - 1)d) = 3000$$

$$\therefore S_{6n} = 1000$$

8. Which of following is a tautology

(1) $((p \rightarrow q) \wedge \sim q) \rightarrow p \wedge q$

(2) $((p \rightarrow q) \wedge \sim q) \rightarrow p$

(3) $((p \rightarrow q) \wedge \sim q) \rightarrow q$

(4) $((p \rightarrow q) \wedge \sim q) \rightarrow \sim q$

Ans. (4)

Sol. (A) $\{(\sim p \vee q) \wedge \sim q\} \rightarrow (p \wedge q)$

$$= (\sim p \wedge \sim q) \rightarrow (p \wedge q)$$

$$= \sim(p \vee q) \rightarrow (p \wedge q)$$

$$= (p \vee q) \vee (p \wedge q)$$

$$= (p \vee q)$$

(B) $(p \vee q) \vee p \equiv p \vee q$

(C) $(p \vee q) \vee q \equiv p \vee q$

(D) $(p \vee q) \vee \sim p \equiv t$

9. If system of equation $4x - \lambda y + 2z = 0$, $2x + 2y + z = 0$, $\mu x + 2y + 3z = 0$, has non trivial solution than.

(1) $\lambda = 6, \mu = 2$

(2) $\mu = 6, \lambda \in \mathbb{R}$

(3) $\mu = 5, \lambda \in \mathbb{R}$

(4) None of these

Ans. (2)

Sol.
$$\begin{vmatrix} 4 & -\lambda & 2 \\ 2 & 2 & 1 \\ \mu & 2 & 3 \end{vmatrix} = 0$$

$$\Rightarrow 4(4) + 1(6 - \mu) + 2(4 - 2\mu) = 0$$

$$\Rightarrow 16 + 6\lambda - \lambda\mu + 8 - 4\mu = 0$$

$$\Rightarrow \lambda = 6, \mu \in \mathbb{R}$$

10. Find the area bounded by the curve $4y^2 = x^2(x - 4)(2 - x)$

(1) $\frac{3\pi}{8}$

(2) $\frac{\pi}{4}$

(3) $\frac{3\pi}{2}$

(4) $\frac{3\pi}{4}$

Ans. (3)

Sol. $4y^2 = x^2(x - 4)(2 - x)$

is defined for $x \in [2, 4] \cup \{0\}$

$$2|y| = |x|\sqrt{(x-4)(2-x)} = |x|\sqrt{-x^2 + 6x - 8}$$

$$x \geq 0$$

$$2|y| = x\sqrt{1 - (x-3)^2}$$

$$A = -2 \int_2^4 \frac{x}{2} \sqrt{1 - (x-3)^2} dx$$

$$= - \int_2^4 (6 - 2x - 6) \sqrt{1 - (x-3)^2} dx = - \int_2^4 (6 - 2x) \sqrt{1 - (x-3)^2} dx + 3 \int_2^4 \sqrt{1 - (x-3)^2} dx$$

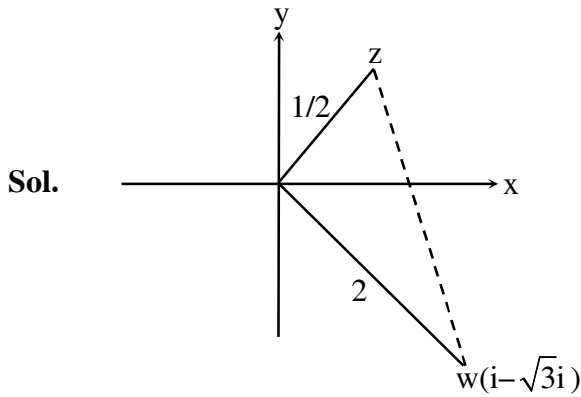
$$= \left[\left(\frac{2}{3} ((x-4)(2-x))^{3/2} \right)_2^4 + 3 \frac{x-3}{2} \sqrt{(x-4)(2-x)} + \frac{1}{2} \sin^{-1}(x-3) \right]_2^4$$

$$A = \frac{3\pi}{2}$$

11. Let $\omega = 1 - \sqrt{3}i$; $|z\omega| = 1$ and $\arg(z) - \arg(\omega) = \frac{\pi}{2}$, then area of the triangle formed by the points w, z and origin is

- (1) $\frac{1}{4}$ (2) $\frac{1}{2}$ (3) 1 (4) 2

Ans. (2)



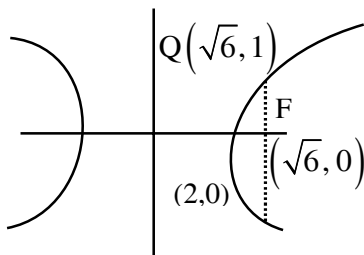
$$\begin{aligned} \text{Area of } \Delta woz &= \frac{1}{2} \times 2 \times \frac{1}{2} \\ &= \frac{1}{2} \end{aligned}$$

12. Let the curve is $x^2 - 2y^2 = 4$. Tangent drawn at $P(4, \sqrt{6})$ cuts the x-axis at R. and latus rectum at $Q(x_1, y_1)$ ($x_1 > 0$), F be focus nearest to P. Area of ΔQPF

- (1) $2 - \frac{\sqrt{6}}{7}$ (2) $2 - \frac{\sqrt{6}}{2}$ (3) $\frac{3}{2}$ (4) $\frac{\sqrt{6}}{2}$

Ans. (2)

Sol. $\frac{x^2}{4} - \frac{y^2}{2} = 1$



$$e^2 = 1 + \frac{2}{4} = \frac{3}{2}$$

$$\Rightarrow e = \sqrt{\frac{3}{2}}$$

$$F(\sqrt{6}, 0), Q(\sqrt{6}, 1), P(4, \sqrt{6})$$

$$\text{Area } (\Delta QPF) = \frac{4 - \sqrt{6}}{2}$$

13. If $\frac{dy}{dx} = (y+1) \left[(y+1)e^{\frac{x^2}{2}} - x \right]$, $y(0) = 2$, then $y'(1)$ is equal to

- (1) $\frac{15}{4\sqrt{e}}$ (2) $\frac{13}{2\sqrt{e}}$ (3) $\frac{15}{7\sqrt{e}}$ (4) $\frac{17}{4\sqrt{e}}$

Ans. (1)

Sol. $\frac{dy}{dx} + (y+1)x = (y+1)^2 e^{\frac{x^2}{2}}$

$$(y+1)^{-2} \frac{dy}{dx} + (y+1)^{-1} x = e^{\frac{x^2}{2}}$$

Put $(y+1)^{-1} = z \Rightarrow -\frac{dy}{(y+1)^2} = dz$

$$\Rightarrow -\frac{dz}{dx} + zx = e^{\frac{x^2}{2}} \Rightarrow \frac{dz}{dx} + (-x)z = -e^{\frac{x^2}{2}}$$

I.F. = $e^{\int -x dx} = e^{-\frac{x^2}{2}}$

Solution is $\left(e^{-\frac{x^2}{2}} \right) z = \int -e^{\frac{x^2}{2}} \times e^{-\frac{x^2}{2}} dx$

$$\Rightarrow \left(e^{-\frac{x^2}{2}} \right) (y+1)^{-1} = -x + c$$

$$\frac{e^0}{2+1} = -0 + c \Rightarrow c = \frac{1}{3}$$

at $x = 1$, $\frac{e^{-\frac{1}{2}}}{y+1} = -1 + \frac{1}{3} = -\frac{2}{3}$

$$\Rightarrow \text{at } x = 1, y + 1 = -\frac{3}{2} e^{-\frac{1}{2}}$$

Now $\left(-\frac{3}{2} e^{-\frac{1}{2}} \right)^{-2} y'(1) + \left(-\frac{2}{3} \right) e^{1/2} = e^{1/2}$

$$\Rightarrow \frac{4}{9} e y'(1) = \frac{5}{3} e^{1/2} \Rightarrow y'(1) = \frac{15}{4\sqrt{e}}$$

14. The term independent of x in expansion of $\left(\frac{x+1}{x^{2/3} - x^{1/3} + 1} - \frac{x-1}{x - x^{1/2}} \right)^{10}$ is :

- (1) 4 (2) 120 (3) 210 (4) 310

Ans. (3)

Sol. $\left((x^{1/3} + 1) - \left(\frac{\sqrt{x} + 1}{\sqrt{x}} \right) \right)^{10}$

$$(x^{1/3} - x^{-1/2})^{10}$$

$$T_{r+1} = {}^{10}C_r (x^{1/3})^{10-r} (-x^{-1/2})^r$$

$$\frac{10-r}{3} - \frac{r}{2} = 0 \Rightarrow 20 - 2r - 3r = 0$$

$$\Rightarrow r = 4$$

$$T_5 = {}^{10}C_4 = \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} = 210$$

15. Let A and B are two square matrix of order n. A relation R is defined such that

$R = \{(A, B) | A = P^{-1}BP \text{ for some invertible matrix } P\}$, then R is

- (1) equivalence (2) reflexive only
 (3) symmetric only (4) transitive only

Ans. (1)

Sol. for reflexive

$$(A, A) \in R \Rightarrow A = P^{-1}AP$$

which is true for $P = I$

\therefore reflexive

for symmetry

As $(A, B) \in R$ for matrix P

$$A = P^{-1}BP \Rightarrow PA = PP^{-1}BP \Rightarrow PAP^{-1} = IBPP^{-1}$$

$$\Rightarrow PAP^{-1} = IB I \Rightarrow PAP^{-1} = B \Rightarrow B = PAP^{-1}$$

$$\therefore (B, A) \in R \text{ for matrix } P^{-1} \quad \therefore R \text{ is symmetric}$$

for transitivity

$$A = P^{-1}BP \text{ and } B = P^{-1}CP \Rightarrow A = P^{-1}(P^{-1}CP)P$$

$$\Rightarrow A = (P^{-1})^2 CP^2 \Rightarrow A = (P^2)^{-1} C(P^2)$$

$$\therefore (A, C) \in R \text{ for matrix } P^2 \quad \therefore R \text{ is transitive}$$

so R is equivalence

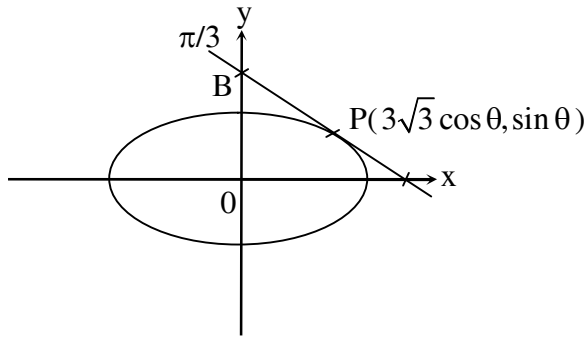
16. If tangent at a point $(3\sqrt{3}\cos\theta, \sin\theta)$ on a curve $\frac{x^2}{27} + y^2 = 1$ cuts intercept on coordinate axis then

find the value of ' θ ' for which the sum of intercept is minimum

- (1) $\frac{\pi}{2}$ (2) $\frac{\pi}{3}$ (3) $\frac{\pi}{6}$ (4) $\frac{\pi}{4}$

Ans. (3)

Sol.



Equation of tangent

$$\frac{x}{3\sqrt{3}} \cos\theta + y \sin\theta = 1$$

$$A\left(\frac{3\sqrt{3}}{\cos\theta}, 0\right), B\left(0, \frac{1}{\sin\theta}\right)$$

Now sum of intercept = $\frac{3\sqrt{3}}{\cos\theta} + \frac{1}{\sin\theta}$

$$y = 3\sqrt{3} \sec\theta + \operatorname{cosec}\theta$$

$$y' = 3\sqrt{3} \sec\theta \tan\theta - \operatorname{cosec}\theta \cot\theta$$

$$y' = 0 \Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$$

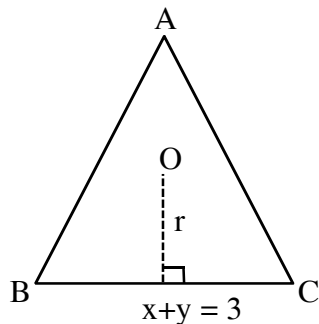
$$\theta = \frac{\pi}{6}$$

17. Centroid of an equilateral ΔABC is origin and one of its side is $x + y = 3$ then the value of $R + r$ is

- (1) $\frac{9}{\sqrt{2}}$ (2) $\frac{7}{\sqrt{2}}$ (3) $\frac{5}{\sqrt{2}}$ (4) $5\sqrt{2}$

Ans. (1)

Sol. $r = \frac{3}{\sqrt{2}}$



$$R = 2r$$

$$R + r = 3r = \frac{9}{\sqrt{2}}$$

18. Let $f : \mathbb{R} - \{3\} \rightarrow \mathbb{R} - \{1\}$; $f(x) = \frac{x-2}{x-3}$ and $g : \mathbb{R} \rightarrow \mathbb{R}$; $g(x) = 2x - 3$ and $f^{-1}(x) + g^{-1}(x) = \frac{13}{2}$.

Then sum of all values of x is

- (1) 2 (2) 3 (3) 5 (4) 7

Ans. (3)

Sol. $f^{-1}(x) + g^{-1}(x) = \frac{13}{2}$

$$\Rightarrow \frac{3x-2}{x-1} + \frac{x+3}{2} = \frac{13}{2}$$

$$\Rightarrow 2(3x-2) + (x-1)(x+3) = 13(x-1)$$

$$\Rightarrow x^2 - 5x + 6 = 0$$

$$\Rightarrow x = 2 \text{ or } 3$$

19. If $x dy - y dx = \sqrt{x^2 - y^2} dx$, $y(1) = 0$ and the area bounded by $y = f(x)$, $y = 0$, $x = 1$ and $x = e^\pi$ is $(\alpha e^{2\pi} + \beta)$ find $10(\alpha + \beta)$.

Ans. 4.0

Sol. $x dy - y dx = \sqrt{x^2 - y^2} dx \Rightarrow \frac{x dy - y dx}{x^2} = \frac{1}{x} \sqrt{1 - \frac{y^2}{x^2}} dx \Rightarrow \int \frac{d\left(\frac{x}{y}\right)}{\sqrt{1 - \left(\frac{y}{x}\right)^2}} = \int \frac{dx}{x}$

$$\Rightarrow \sin^{-1}\left(\frac{y}{x}\right) = \ln|x| + c$$

at $x = 1, y = 0 \Rightarrow c = 0$

$$y = x \sin(\ln x)$$

$$A = \int_1^{e^\pi} x \sin(\ln x) dx$$

$$x = e^t, dx = e^t dt = \int_0^\pi e^{2t} \sin(t) dt$$

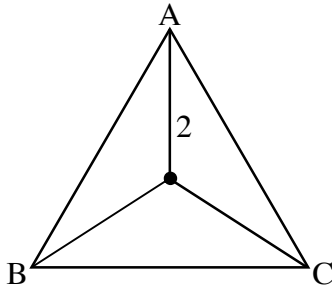
$$\alpha e^{2\pi} + \beta = \left(\frac{e^{2t}}{5} (2 \sin t - \cos t) \right)_0^\pi = \frac{1 + e^{2\pi}}{5}$$

$$\alpha = \frac{1}{5}, \beta = \frac{1}{5} \text{ so } 10(\alpha + \beta) = 4$$

20. In a ΔABC whose circum radius is 2. A pole standing inside the ΔABC and angle of elevation of top of the pole from points A, B, C is 60° then find height of pole

Ans. 3.464

Sol.



$$\tan 60^\circ = \frac{h}{2} \Rightarrow h = 2\sqrt{3}$$

21. Let $Q(x) = f(x^3) + xg(x^3)$ and $Q(x)$ is divisible by $x^2 + x + 1$, then find value of $Q(1)$

Ans. 0

Sol. roots of $x^2 + x + 1$ are ω and ω^2 now

$$Q(\omega) = f(1) + \omega g(1) = 0$$

$$Q(\omega^2) = f(1) + \omega^2 g(1) = 0$$

$$\text{add} \Rightarrow 2f(1) - g(1) = 0 ; g(1) = 2f(1) \Rightarrow f(1) = g(1) = 0$$

$$Q(1) = f(1) + g(1) = 0 + 0 = 0$$

22. If $P = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}$ and $P^n = 5I = 8P$ then value of n is

Ans. 6

Sol.
$$P^2 = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ -5 & 4 \end{bmatrix}$$

$$P^4 = \begin{bmatrix} -1 & 1 \\ -5 & 4 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ -5 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 3 \\ -15 & 11 \end{bmatrix}$$

$$P^6 = \begin{bmatrix} -4 & 3 \\ -15 & 11 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ -5 & 4 \end{bmatrix} = \begin{bmatrix} -11 & 8 \\ -40 & 29 \end{bmatrix}$$

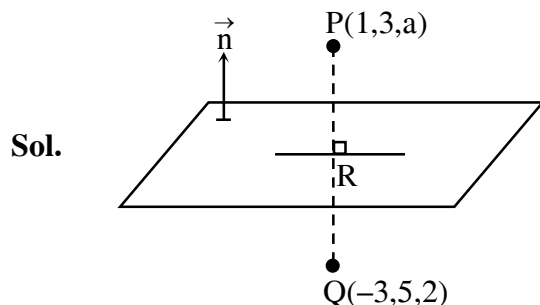
and

$$5I - 8P = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - 8 \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} -11 & 8 \\ -40 & 29 \end{bmatrix}$$

So $n = 6$

23. Image of point $(1, 3, a)$ in the plane $\vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) - b = 0$ is $(-3, 5, 2)$ find $|a + b|$

Ans. 1



plane : $2x - y + z = b$

$R \equiv \left(-1, 4, \frac{a+2}{2}\right) \rightarrow$ on plane

$\therefore -2 - 4 + \frac{a+2}{2} = b$

$\Rightarrow a + 2 = 2b + 12 \Rightarrow a = 2b + 10 \dots\dots (i)$

$\langle PQ \rangle = \langle 4, -2, a - 2 \rangle$

$\therefore \frac{2}{4} = \frac{-1}{-2} = \frac{1}{a-2} \Rightarrow a - 2 = 2 \Rightarrow a = 4, b = -3$

$\therefore |a + b| = 1$

24. Let $f(x) = \begin{cases} \frac{\sin(a+1)x + \sin(2x)}{2(x)} & x < 0 \\ b & x = 0 \\ \frac{\sqrt{x - bx^3} - \sqrt{x}}{bx^{5/2}} & x > 0 \end{cases}$, $f(x)$ is continuous at $x = 0$, then $|a + b|$ is equal

Ans. 4.5

Sol. LHL = $\frac{(a+1)}{2} + 1 = b = f(0)$

$\Rightarrow 2b = a + 3 \dots\dots(i)$

RHL = $\lim_{x \rightarrow 0^+} \frac{\sqrt{1 - bx^2} - 1}{bx^2}$

= $\lim_{x \rightarrow 0^+} \frac{-bx^2}{bx^2(\sqrt{1 - bx^2} + 1)}$

= $-\frac{1}{2}$

$\therefore b = -\frac{1}{2} \Rightarrow a = -4$

$|a + b| = \frac{9}{2}$

25. If $\sum_{k=0}^{10} (2^k + 3)^{10} C_k = \alpha \cdot 2^{10} + \beta \cdot 3^{10}$ then value of $\alpha + \beta$

Ans. 4

Sol. $\sum_{k=0}^{10} {}^{10}C_k 2^k + \sum_{k=0}^{10} 3 \cdot {}^{10}C_k$
 $3^{10} + 2 \cdot 2^{10} \Rightarrow \alpha = 3, \beta = 1 \Rightarrow \alpha + \beta = 4$

26. Let $f(x)$ is a differentiable function $\forall x \in \mathbb{R}$ such that $f(x + y) = f(x) \cdot f(y)$ and $f'(0) = 3$ then

$\lim_{x \rightarrow 0} \frac{f(x) - 1}{x}$ is equal to

Ans. 3

Sol. If $f(x + y) = f(x) \cdot f(y)$ & $f'(0) = 3$ then

$$f(x) = a^x \Rightarrow f'(x) = a^x \ln a$$

$$\Rightarrow f'(0) = \ln a = 3 \Rightarrow a = e^3$$

$$\Rightarrow f(x) = (e^3)^x = e^{3x}$$

$$\lim_{x \rightarrow 0} \frac{f(x) - 1}{x} = \lim_{x \rightarrow 0} \left(\frac{e^{3x} - 1}{3x} \times 3 \right) = 1 \times 3 = 3$$

27. Let $f(x)$ be a cubic polynomial such that it has maxima at $x = -1$, minima at $x = 1$, $\int_{-1}^1 f(x) dx = 18$,

$f(2) = 10$ find sum of coefficient of $f(x)$.

Ans. 8

Sol. $f'(x) = k(x+1)(x-1)$

$$\therefore f(x) = \frac{kx^3}{3} - kx + C$$

$$f(2) = 10 = \frac{8k}{3} - k + C \quad \dots(i)$$

$$\int_{-1}^1 f(x) dx = 18 \Rightarrow \int_{-1}^1 \left(k \left(\frac{x^3}{3} - x \right) + C \right) dx = 18$$

$$\Rightarrow 0 + 2C = 18 \Rightarrow C = 9$$

$$\therefore 1 = \frac{2k}{3} \Rightarrow k = \frac{3}{2} \quad \dots(\text{from (i)})$$

$$\therefore f(x) = \frac{x^3}{2} - \frac{3}{2}x + 9$$

Sum of co-efficient = $-1 + 9 = 8$